Notes on the Fermi Liquid Theory

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Contents

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Historically speaking, the Fermi liquid theory was developed by Landau to explain the properties of liquid ^{3}He [LD Landau, “On the theory of the Fermi liquid”, Sov. Phys. JETP 8.1 (1959), p. 70.], but soon it was expanded to a theory explaining the behaviour of normal metals with a weak electron-electron interaction. The theory is a natural expansion of a non-interacting system of fermions, usually called a Fermi-gas, to a system in which we gradually introduce an interaction, namely, a Fermi liquid.

Landau think about this problem like this: one can slowly introduce the interaction between the fermions, and following the adiabatic theorem, the ground state of the Fermi gas would transform into the ground state of the interacting system. In this process, the spin, charge and momentum of the fermions, remain unchanged, while their dynamical properties, such as their mass, magnetic moment etc. are renormalized to a new set of values. This was the basic idea of Landau behind the theory of Fermi liquids. Landau introduced a quasiparticle excitations in the new interacting theory, corresponding to the Fermi particles of the Fermi gas. This idea makes the liquid theory qualitatively similar to the Fermi gas theory, which was something that was observed experimentally for the liquid state of ^{3}He. This theory, even though it was initially aimed to explain the similarities and differences seen between the liquid phase of ^{3}He and the non-interacting gas, it was soon after realized that it could be a good model for conduction electrons inside a normal metal, since they can also be seen as weakly interacting fermions. This theory is usually preferred over some of the basic and famous single-electron theories such as Hartree-Fock approximation, since it is not trying to explain the interaction using Slater determinants.

ADIABATIC CONTINUITY AND QUASIPARTICLES

The physical picture is the following. In the non-interacting system, the spectrum consists of particles and holes with a certain dispersion (the free one-particle spectrum) $\epsilon_0(p)$. These excitations are infinitely long-lived since they cannot decay in the absence of matrix elements (interactions). The ground state is characterized by a distribution function $n_0(p)$.

Landau reasoned that what interactions do is to produce (virtual) particle-hole pairs. Thus, the distribution function must change: $n_0(\mathbf{p}) \to n(\mathbf{p})$. He further assumed that this change is a smooth function (i.e. analytic) of the interaction. In other words, as the interactions are slowly turned on, the non-interacting states are assumed to smoothly

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Non-interacting Fermi gas	(Weakly) Interacting Fermi liquid
electron $\|p > p_F\|$	quasiparticle $\|p > p_F\|$
hole $\|p < p_F\|$	quasihole $\|p < p_F\|$
quantum number (e.g. spin 1/2)	quantum number (e.g. spin 1/2)
energy dispersion $\epsilon_0(p)$	renormalized energy $\epsilon(p)$
density distribution $n_0(p)$	density distribution $n(p)$
mass m	effective mass m^*
always stable $\tau = \infty$	finite lifetime τ

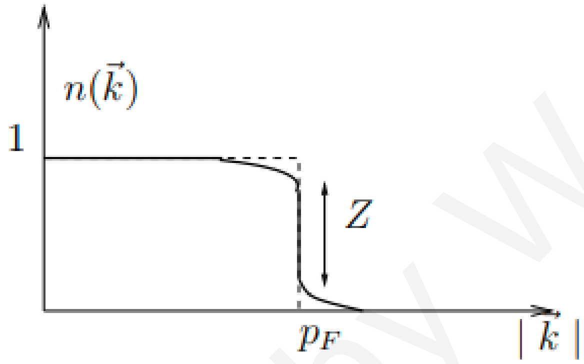

FIG. 1: Discontinuity in the occupation number at the Fermi surface in a non-interacting (dashed) and in an interacting system (solid).

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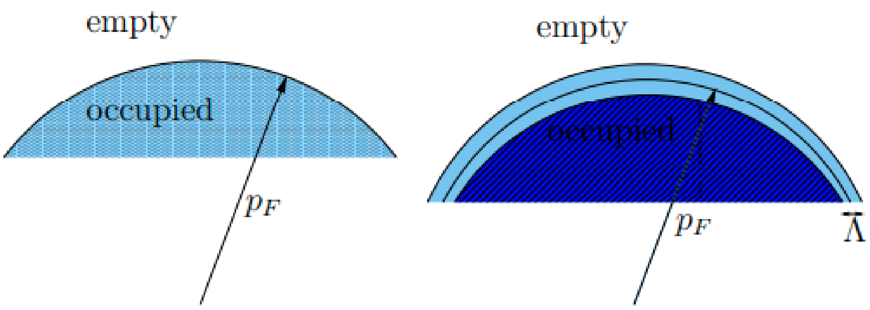
and continuously evolve into interacting states, and that this evolution takes place without hitting any singular behavior. Such a singularity would signal an instability of the ground state and should be viewed as a phase transition. Thus, if there are no phase transitions, there should be a smooth connection between noninteracting and interacting states. In particular the quantum numbers used to label the non-interacting states should also be good quantum numbers in the presence of interactions. The one-electron (particle) state becomes a quasiparticle which carries the same momentum k and spin ($\pm 1/2$) of the bare electron.

The state of the interacting systems can be parameterized by the actual distribution function $n(\mathbf{p})$. Let $\delta n(\mathbf{p}) \equiv n(\mathbf{p}) - n_0(\mathbf{p})$. For the system to be stable $\delta n(p)$ must be non-zero only for $|p| \approx p_F$ and the ground state energy is determined by $\delta n(\mathbf{p})$. If $n_0(\mathbf{p}) \rightarrow n(\mathbf{p}) = n_0(\mathbf{p}) + \delta n(\mathbf{p})$ with $\delta n/n \ll 1$(for all p close to the fermi surface), the total energy



(a) Non-interacting system (b) Interacting system

FIG. 2: Fermi surface in a) non-interacting system and b) in an interacting system. Only the states in the narrow region $\Lambda \ll p_F$ contribute in the presence of interactions.

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is $E = E_0 + \delta E$ and we can expand the excitation energy δE in powers of the change of the distribution function $\delta n(p)$ as

$$\delta E = \sum_{\mathbf{p}} \epsilon(\mathbf{p})\delta n(\mathbf{p}) + ... \tag{1}$$

The excitation energy δE should also tell us how much energy does it cost to add an excitation of momentum p close to the Fermi surface. Thus, the quasiparticle energy should be

$$\epsilon(\mathbf{p}) = \frac{\delta E}{\delta n(\mathbf{p})} \tag{2}$$

The difference between the energy of the ground state with $N+1$ particles and N particles is the chemical potential μ:

$$\mu = E(N+1) - E(N), \tag{3}$$

$$\mu = \epsilon(p)|_{p=p_F} \tag{4}$$

The higher order corrections to δE in powers of $\delta n(p)$ gives the interactions among quasiparticles. This is the Landau expansion:

$$E = \sum_{p} \epsilon_0(p)\delta n(p) + \sum_{p,p'} f(p,p')\delta n(p)\delta n(p') + O((\delta n(p))^3) \tag{5}$$

where we have introduced the Landau parameters, the symmetry function $f(p,p') = f(p',p)$.

Now we get the the quasiparticle energy should be

$$\epsilon(p) = \frac{\delta E}{\delta n(p)} = \epsilon_0(p) + \sum_{p'} f(p,p')\delta n(p') + ... \tag{6}$$

The correction term gives a measure of the change of the quasiparticle energy due to the presence of other quasiparticles. The function $f(p,p')$ measures the strength of quasiparticle-quasiparticle interactions, which is called *Landau parameters.* Hence $f(p,p')$ is an effective interaction for excitations arbitrarily close to the Fermi surface.

Near the Fermi surface, the Fermi velocity is given by $v_F(p) = \nabla\epsilon(p)$. Hence we can define the effective mass

$$m^* = \frac{p_F}{|v_F(p_F)|} \tag{7}$$

At finite temperature, the distribution function will be

$$n(p) = \frac{1}{\exp[\beta(\epsilon(p)-\mu)]+1} \tag{8}$$

This equation is actually a self-consistency equation for $n(p)$, since $\epsilon(p)$ depends on $n(p)$.

Likewise, the interactions between quasiparticles depends only on the relative orientation of the spins σ and σ'. The Landau interaction term is modified by spin effects as $\sum_{p,p'} f(p,p')\delta n(p)\delta n(p') \to \sum_{p,p'}\sum_{\sigma,\sigma'} f_{\sigma\sigma'}(p,p')\delta n_\sigma(p)\delta n_{\sigma'}(p')$. The general expression gives

$$f_{\sigma\sigma'}(p,p') = f(p,p') + 4\sigma\sigma'\phi(p,p') \tag{9}$$

For a rotationally invariant system, the interaction functions must depend only on the angle θ: $\cos\theta(p,p') = \frac{p\cdot p'}{p_F^2}$. Here we should be able to use the expansion

$$f_{pp'} = \sum_{l=0}^{\infty} f_l P_l(\cos\theta), \quad \phi_{pp'} = \sum_{l=0}^{\infty} \phi_l P_l(\cos\theta) \tag{10}$$

$$f_l = \frac{2l+1}{2}\int_0^1 dx P_l(x) f(p.p'), \quad \phi_l = \frac{2l+1}{2}\int_0^1 dx P_l(x)\phi(p.p') \tag{11}$$

which is the definition of the Landau parameters.

Effective mass

The net momentum of quasiparticles is

$$P_{qp} = \int \frac{d^3p}{(2\pi^3)} p n_p \tag{12}$$

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which is also the momentum of the Fermi liquid. Since the momentum is just the particle mass times this current

$$P_p = m\int \frac{d^3p}{(2\pi^3)} v_p n_p \tag{13}$$

which leads to

$$\int \frac{d^3p}{(2\pi^3)} p n_p = m\int \frac{d^3p}{(2\pi^3)} \nabla_p \epsilon(p) n_p \tag{14}$$

Now make an arbitrary change of $n(p)$ and recall that $\epsilon(p)$ depends upon $n(p)$, so that

$$\delta\epsilon(p) = \int \frac{d^3p'}{(2\pi^3)} f(p,p')\delta n(p') \tag{15}$$

We have

$$\begin{aligned}
\int \frac{d^3p}{(2\pi^3)} p\delta n_p &= m\int \frac{d^3p}{(2\pi^3)} \nabla_p\epsilon(p)\delta n_p + m\int \frac{d^3p}{(2\pi^3)} \int \frac{d^3p'}{(2\pi^3)} \nabla_p (f(p,p')\delta n(p')) n_p && (16)\\
&= m\int \frac{d^3p}{(2\pi^3)} \nabla_p\epsilon(p)\delta n_p - m\int \frac{d^3p}{(2\pi^3)} \int \frac{d^3p'}{(2\pi^3)} f(p,p')\delta n(p)\nabla_{p'} n_{p'} && (17)\\
\Rightarrow \frac{\mathbf{p}}{m} &= \nabla_p\epsilon(p) - \int \frac{d^3p'}{(2\pi^3)} f(p,p')\nabla_{p'} n(p') && (18)
\end{aligned}$$

The factor $\nabla_p n(p) = \nabla_p\epsilon(p)\nabla_\epsilon n = -\frac{\mathbf{p}}{m}\delta(\epsilon_p - E_F)$. The integral may be evaluated by taking advantage of the system isotropy, and setting p parallel to the z-axis, on the Fermi surface

$\nabla_p\epsilon(p)|_{p_F} = p_F/m^*$. Thus,

$$\begin{aligned}
\frac{p_F}{m} &= \frac{p_F}{m^*} + \int \frac{d^3p'}{(2\pi)^3} f(p,p')\frac{p_F}{m}\delta(\epsilon_p - E_F) && (19)\\
\frac{1}{m} &= \frac{1}{m^*} + \frac{1}{m}\frac{\int dp'(p')^2 \int d\theta 2\pi}{(2\pi)^3} f(\cos\theta_{p_F,p'})\cos\theta_{p_F,p'}\delta(\epsilon_p - E_F) && (20)\\
\frac{1}{m} &= \frac{1}{m^*} + N(E_F)\frac{1}{m}\frac{\int d\theta 2\pi}{(2\pi)^2} f(\cos\theta_{p_F,p'})\cos\theta_{p_F,p'} && (21)
\end{aligned}$$

We expand f as

$$f(\theta) = \sum_l f_l P_l(\cos\theta) \tag{22}$$

$$\int P_n(x)P_m(x)dx = \frac{2}{2l+1}\delta_{nm} \tag{23}$$

and we define

$$N(E_F)f_l = \frac{m^* p_F}{\pi^2} f_l \equiv F_l \tag{24}$$

So we reach that

$$\frac{1}{m} = \frac{1}{m^*} + \frac{F_1}{3m^*} \tag{25}$$

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Scattering rate, quasiparticle life time

Hence the scattering rate is related to the quasi-particle lifetime as

$$\Gamma \sim \frac{1}{\tau} \tag{26}$$

The scattering rate can be estimated by the collision integral from the Fermi's Golden rule:

$$\begin{aligned}\frac{1}{\tau_k} = \int \frac{d^3q}{(2\pi)^3} \int \frac{d^3p}{(2\pi)^3} 2\pi |\langle k-q, p+q|t|k,p\rangle|^2 \delta(E(p)+E(k)-E(k-q)-E(p+q)) \\ \times[n(p)(1-n(p+q))(1-n(k-q)) + (1-n(p))n(p+q)n(k-q)]\end{aligned} \tag{27}$$

The first term in the expression in brackets represents the rate at which quasiparticles are scattered into new unoccupied states while the second term represents the blocking of such processes due to occupied states.

To evalute this formula exactly, we leave it for the more advanced course. [We will derive this express and calculate it in the course of Quantum Many-body Physics]. Here we just make some proper approximatin and get a very rough estimation. The t-matrix amplitude $|\langle k-q, p+q|t|k,p\rangle|^2$ is represented by summing up particle-particle (or particle-hole) scattering processes. We just approximate it as a constant, independent of momentum transfer or energies of quasiparticles. Then we get

$$\begin{aligned}\frac{1}{\tau} &\sim |t|^2 2\pi \int_0^\infty d\xi_{p1} \int_0^\infty d\xi_{p2} \int_0^\infty d\xi_{p3} \rho(\xi_{p1})\rho(\xi_{p2})\rho(\xi_{p3})\delta(\xi_k + \xi_{p3} - \xi_{p1} - \xi_{p2}) \\ &\sim |t|^2 2\pi \int_0^\omega d\xi_{p1} \int_0^\omega d\xi_{p2} \rho(\xi_{p1})\rho(\xi_{p2})\rho(\xi_{p1}+\xi_{p2}-\xi_k) \\ &\sim |t|^2 2\pi \rho^3 \omega^2\end{aligned} \tag{28}$$

Thus, provided the interaction strength and the density of states remain finite in the vicinity of the Fermi surface. is bounded by a constant times ω^2 and the quasiparticle lifetime diverges at the Fermi surface as ω goes to zero. For the finite temperature, one just replaces $\omega \sim k_B T$, so we get

$$\frac{1}{\tau} \sim (k_B T)^2 \tag{29}$$

The Landau picture we discussed works very well in neutral Fermi fluids (such as the normal phase of liquid He) and in most (simple) metals.

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EQUILIBRIUM PROPERTIES OF THE NORMAL FERMI LIQUID

Specific heat

The low temperature specific heat of a Fermi liquid, just as in the case of non-interacting fermions, is linear in T with a coefficient determined by the effective mass m^* of the quasiparticles at p_F. Let's compute the low temperature entropy, or rather the variation of the quasiparticle entropy (per unit volume) as

$$S = -k_B/V \sum_{p,\sigma} [n(p) \ln n(p) + (1 - n(p)) \ln(1 - n(p))] \tag{30}$$

where $n(p)$ is the Fermi-Dirac distribution

$$n(p) = \frac{1}{1 + e^{(\epsilon(p) - \mu)/k_B T}} \tag{31}$$

The variation of the entropy is

$$\delta S = \frac{1}{TV} \sum_p (\epsilon(p) - \mu) \delta n(p) \tag{32}$$

$$\delta n(p) = \frac{\partial n(p)}{\partial \epsilon(p)} [-\frac{\epsilon(p) - \mu}{T} \delta T + \delta\epsilon(p) - \delta\mu] \tag{33}$$

Next we concern with the temperature dependence so we only focus on the first term:

$$\delta S = -\frac{1}{V} \sum_p \frac{\partial n(p)}{\partial \epsilon(p)} (\epsilon(p) - \mu)^2 \frac{\delta T}{T^2} \tag{34}$$

$$= -\int p^2 \frac{dp}{d\epsilon} d\epsilon \frac{\partial}{\partial \epsilon} [\frac{1}{e^{(\epsilon - \mu)/k_B T} + 1}] (\frac{\epsilon - \mu}{T})^2 \delta T \tag{35}$$

$$\approx -k_B^2 N(E_F) \int dx \frac{\partial}{\partial x} (\frac{1}{e^x + 1}) x^2 \delta T \tag{36}$$

$$\approx \frac{\pi^2}{3} N(E_F) k_B^2 T \Rightarrow C_V = T(\frac{\partial S}{\partial T})_V = \frac{\pi^2}{3} N(E_F) k_B^2 T \tag{37}$$

This form is identical to that of a Fermi gas. It should be noted, however, that the ensity of states at the Fermi level, $N(E_F)$, has received renormalization from the renormalization of the Fermi velocity by the interactions. One way to emphasize this fact is to re-write this form in terms of the free-fermion heat capacity $C^{(0)}V(T)$:

$$C_V = C_V^{(0)} \frac{m^*}{m} \tag{38}$$

The DoS at the Fermi level is

$$N(E_F) = \frac{1}{V} \sum_k \delta(E_k - E_F) = \frac{2}{(2\pi)^3} 4\pi \int p^2 dp \delta(\frac{k^2}{2m} - E_F) = \frac{m k_F}{\pi^2} \tag{39}$$

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Compressibility

Compressibility describes the change in density if the chemical potential is changed. For a fixed particle number, we have

$$\kappa^{-1} = n^2 \frac{\partial \mu}{\partial n} \tag{40}$$

where chemical potential is $\mu = \epsilon(k_F)$, n is charge density.

Using Eq. 6 we have

$$\frac{\partial \mu}{\partial n} = \frac{\partial \epsilon^0}{\partial k_F} \frac{\partial k_F}{\partial n} + \int d^3 k' f(k_F, k') \frac{\partial \delta n(k')}{\partial k_F} \frac{\partial k_F}{\partial n} \tag{41}$$

We use the relation $n = \frac{k_F^3}{3\pi^2}$, $\frac{\partial \epsilon^0}{\partial k_F} = \frac{k_F}{m}$ and $\frac{\partial \delta n(k')}{\partial k_F} = \delta(k' - k_F)$, and then obtain

$$n \frac{\partial \mu}{\partial n} = \frac{k_F^2}{3m} + \frac{k_F^3}{3\pi^2} \int d\Omega f(\theta) = \frac{k_F^2}{3m}(1 + F_0) \tag{42}$$

and finally

$$\kappa^{-1} = n \frac{k_F^2}{3m}(1 + F_0) \tag{43}$$

Spin susceptibility

Up to now we discussed only spin-independent perturbations. They are connected to the parameters F_l. Now, we discuss spin-dependent perturbations. The magnetic susceptibility is given by $\chi = \frac{\partial M}{\partial H}$. The magnetisation is given by

$$\chi = \frac{k_F m^*}{4\pi^2 (1 + Z_0)} \tag{44}$$

where $Z_0 = N(E_F) \frac{1}{2} \int_0^1 d(\cos\theta) P_l(\cos\theta)(f_{\uparrow\uparrow}(\theta) - f_{\downarrow\uparrow}(\theta))$ is anti-symmetric Landau parameter.

ZERO SOUND

Taking the quasiparticles in Fermi liquid theory as classical, we can think about its transport properties. A typical feature is the so-called ”zero sound”.

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Similar to Boltzmann transport theory, we write down the kinetic equation for (non-equibrium) distribution function of Fermi liquid

$$\frac{dn}{dt} = \frac{\partial n}{\partial t} + \frac{\partial n}{\partial r}\frac{\partial r}{\partial t} + \frac{\partial n}{\partial p}\frac{\partial p}{\partial t} \tag{45}$$

As we did before, we assume the distribution function taking the form $n(p, r, t) = n_0(p) + \delta n(p, r, t)$. The energy of quasiparticle is

$$\epsilon = \epsilon_0 + \delta\epsilon \tag{46}$$

and

$$\delta\epsilon(p, r, t) = \int d^3p' f(p, p')\delta n(p', r, t) \tag{47}$$

According to the classical mechanics, we have

$$\frac{dr}{dt} = \frac{\partial\epsilon}{dp}, \quad \frac{dp}{dt} = f(r, t) = -\frac{\partial\epsilon}{dr} \tag{48}$$

where the last equation is the rate of change of quasiparticle momentum (the force).

So we have the following steps,

$$\frac{\partial n}{\partial r}\frac{\partial r}{\partial t} = \frac{\partial(n_0 + \delta n)}{\partial r}\frac{\partial\epsilon}{dp} \approx \frac{\partial\delta n}{\partial r}\frac{\partial\epsilon_0}{dp} \tag{49}$$

$$\frac{\partial n}{\partial p}\frac{\partial p}{\partial t} = \frac{\partial(n_0 + \delta n)}{\partial p}(-\frac{\partial\epsilon}{dr}) \approx -\frac{\partial\delta\epsilon}{dr}\frac{\partial n_0}{\partial p} \tag{50}$$

and we reach

$$\frac{dn}{dt} = \frac{\partial\delta n}{\partial t} + \frac{\partial\delta n}{\partial r}\frac{\partial\epsilon_0}{dp} - \frac{\partial\delta\epsilon}{dr}\frac{\partial n_0}{\partial p} = 0 \tag{51}$$

for the collisionless condition.

Here we set the form of distribution function as

$$\delta n = \delta(\epsilon - \epsilon_F)M(\mathbf{p})e^{ikr-\omega t} \tag{52}$$

$M(\mathbf{e})$ is some unknown function. and we note that

$$\frac{\partial\epsilon}{\partial r} = \frac{\partial\delta\epsilon}{\partial r} = \int d^3p' f(p, p')\frac{\partial\delta n(p', r, t)}{\partial r} \tag{53}$$

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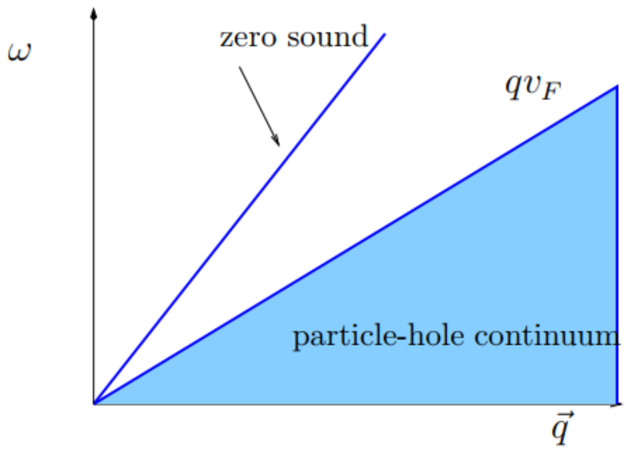

FIG. 3: Spectrum of collective modes.

Inserting the above two equations, we get

$$(\omega - v_F \mathbf{p}\cdot\mathbf{k})M(\mathbf{p}) = (\mathbf{e}\cdot\mathbf{k})v_F \int d^3p' f(p,p')\delta(\epsilon-\epsilon_F)M(\mathbf{p}') \tag{54}$$

$$= (\mathbf{e}\cdot\mathbf{k})\int dp' d\theta d\phi (p')^2 \sin\theta f(p,p')\delta(p-p_F)M(\mathbf{p}') \tag{55}$$

$$= (\mathbf{e}\cdot\mathbf{k})2\pi p_F^2 \int d\theta \sin\theta f(\theta)M(\mathbf{p}') \tag{56}$$

where we used $\frac{\partial n_0}{\partial p} = \frac{\partial n_0}{\partial \epsilon}\frac{\partial \epsilon}{\partial p} = -\mathbf{v}\delta(\epsilon - E_F)$. This is a self-consistent equation to solve $M(\mathbf{p})$. This is very difficult, so next we try some approximation.

If $f(\theta) = F_0$ is a constant (for $l = 0$ channel), then we see the solution ansatz should be

$$M(\mathbf{p}) = c_0 \frac{(\mathbf{e}\cdot\mathbf{k})}{(\omega - v_F \mathbf{p}\cdot\mathbf{k})} \tag{57}$$

Inserting this form to the integral equation, we get

$$\frac{S}{2}\ln\frac{S+1}{S-1} - 1 = F_0^{-1} \tag{58}$$

where we define $S = \omega/(kv_F)$. Importantly, we get

$$S = \begin{cases} 1 + e^{-F_0^{-1}} F_0 \ll 1 \\ \sqrt{\frac{F_0}{3}}, F_0 \gg 1 \end{cases} \tag{59}$$

This solution ($S > 1$) corresponds to an (undamped) sound mode with dispersion $\omega = qv_F S$ and a sound velocity $c_0 = v_F S$. This collective mode is known as zero sound. Notice that the edge of the particle-hole continuum is at $\omega = qv_F$. Zero sound mode has been observed in ^{3}He experiment in the regime between 100 mK and 3 mK .

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INSTABILITY

As discussed in the previous sections, Landau Fermi-liquid theory is based on the assumption that, as the interaction is turned on slowly, the free-fermion ground state evolves adiabatically to the interacting ground state. It can occur, however, that interactions make the Fermi-liquid ground state unstable, and the system goes through a quantum phase transition into a new phase (often with spontaneously broken symmetry). There are several different types of Fermi-liquid instabilities.

Ferromagnet.— Equation 44 suggests that the spin susceptibility χ diverges as $Z_0 \to -1$. This indicates the ferromagnetic instability, as the system responds infinitely strongly to a weak Zeeman field that couples to the electron spins. The ferromagnetic instability corresponds to a quantum phase transition in which the magnetization increases to a non-zero value.

Pairing Instability.— Discuss soon.

Charge Density-Wave Instabilities.— Think about one dimension that the Fermi surface reduces to a pair of discrete Fermi points. $2k_F$ scattering will lead to the Peries instability and form the charge density wave state. We are thus led to the conclusion that the 1D Fermi liquid is unstable against any interaction; the only Fermi-liquid state is a free Fermi gas in 1D. Similarly, in 2D, nested Fermi surface may drive the instability of charge density wave state.

CONCLUDING REMARKS

After all the vagueness of the derivation in this chapter, it is perhaps difficult to imagine that the Fermi liquid theory is able to describe anything. But it actually is: it gives a good description for instance of the ^{3}He quantum liquid, as well as predicts many basic features of metals. However, there are important states of fermionic matter that cannot be described by the Fermi liquid theory. For instance superconductivity/superfluidity that will be discussed in the following chapter cannot be explained using the Fermi liquid theory. The concept of a quasiparticle will be used also there, but its meaning is different from the Fermi liquid context. Since the Fermi liquid theory, the concept of “quasiparticle” becomes a cornerstone of solid state physics, with the well known examples including phonon, exciton, plasmon,

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magnon, spinon, just to name a few.

Furthermore, one may wonder the situation that the Fermi liquid theory fails. People are used to call non-Fermi liquid in this case. One example is, in one-dimensional systems the Fermi liquid theory always fails. There, a somewhat analogous description is given by the Luttinger liquid theory. Fermi liquid theory can be considered as the 'default', basic description that one should try to apply for an interacting fermion system to start with. Often it applies pretty well; if not, one probably has a highly interesting and non-trivial many-body system at hand.

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HOMEWORK

1. Please derive the spin susceptibility Eq. 44 using the Fermi liquid theory.

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[1] LD Landau, “On the theory of the Fermi liquid”, Sov. Phys. JETP 8.1 (1959), p. 70.

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